Complex Regge Poles in the Cut j Plane: Pion-Nucleon Charge-Exchange Scattering*

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The structure of the pole-cut combination in πN charge-exchange scattering for $t \leq 0$ is represented by a pair of complex poles. Excellent fits to the πN charge-exchange differential cross section, polarization, and total cross-section difference $\sigma_t(\pi^- p)-\sigma_t(\pi^+ p)$ are obtained for the two examples considered, $\mathrm{Im}\alpha = g\sqrt{-t}$ and $\mathrm{Im}\alpha = g$. The phases of the residues are allowed to vary and a zero in $\mathrm{Im}A'^{(-)}$ (the "crossover" zero) is obtained.

Within the past few years a substantial amount of theoretical and experimental evidence has been accumulated which indicates that cuts in the angular momentum plane are important. It is generally recognized that such Regge cuts arise from the combination of two or more Regge-pole exchanges, that the dominant cut in a given channel results from the combination of the dominant Regge pole in that channel and the Pomeranchon, that the cuts lie along the negative real axis to the left of $j=1$, and that the collision of a pole and its dominant cut occurs at some t in the neighborhood of $t=0$. However, unlike the situation in the energy plane, the precise nature and the magnitude of the j-plane singularities are unknown. In fact, one does not even know whether, for $t<0$, a t-channel Regge pole lies on the physical or the unphysical sheet of the j_t plane. It may be noted that in the energy plane the (resonance) poles are on the unphysical sheet because of the causality principle. No such principle helps in the j plane.

There have been several models proposed to describe Regge cuts. To name some of them, there are the Amati-Fubini-Stanghellini,[1] the multi-Regge,[2] the Regge-eikonal,[3] the absorptive,[4] the Gribov-Migdal,[5] and the Carlitz-Kislinger models.[6] Under various approximations the singularity at the branch point in the above models is logarithmic except for the last one where it is square root. The difficulty in all these models is that of correctly implementing unitarity conditions at high energies. It is quite conceivable that if and when this difficulty is removed, all or some of the models may turn out to be different approximate versions of the same basic theory. Nevertheless, as things stand now, we are faced with different models with different predictions.

It is, therefore, of interest to discuss Regge cuts within a formalism which is free, as far as possible, of any ad hoc assumptions associated with any specific model but which, at the same time, incorporates all the basic consequences of the cut j plane. Recently Kaus and Zachariasen[7] have noted that the existence of a pole in a given sheet generally implied poles in all the sheets and thus they have shown that as a Regge pole on the physical sheet moves towards the branch point, when t is decreased below the t-channel threshold, one of two things happens: (a) It is met by a pole which comes from the nearby unphysical sheet through the branch point, and the two then continue their motion in the physical sheet as a complex conjugate pair; or (b) the pole goes through the branch point and meets the pole in the nearby unphysical sheet, and the two then continue to move in the unphysical sheet as a complex-conjugate pair. At high energies, then, an amplitude, in the case (a), is represented by the pair of complex-conjugate poles plus the cut, whereas in case (b), it is represented by the cut alone.

Furthermore, Ball, Marchesini, and Zachariasen[8] have shown that the cut contribution itself can be well approximated, from moderate to rather high energies, by a pair of complex-conjugate poles alone. The positions of the poles are identical to the poles discussed above, and they have complex-conjugate residues that depend on the strength of the cut. Thus one can express a given amplitude as

$$T(s,t)=\gamma_+\frac{(1-e^{-i\pi\alpha_+})}{\sin\pi\alpha_+}\left(\frac{s}{s_0}\right)^{\alpha_+}+\gamma_-\frac{(1-e^{-i\pi\alpha_-})}{\sin\pi\alpha_-}\left(\frac{s}{s_0}\right)^{\alpha_-}, \tag{1}$$

where the signature factor has been explicitly taken out and

$$\alpha_+=\alpha_-^*=\mathrm{Re}\alpha+i\,\mathrm{Im}\alpha,\quad \gamma_+=\gamma_-^*=|\gamma|e^{i\varphi}. \tag{2}$$

For case (a), $\gamma_\pm$ is a sum of the contributions from the (physical sheet) poles and the cut, and for case (b), $\gamma_\pm$ contains only the cut contribution. Models for $\alpha_\pm(t)$ are discussed in Ref. 7 in terms of t- and j-plane analyticity. They differ primarily by the number of sheets in the j plane (ranging from two[6] to infinity[2]). Regardless of the number of sheets (and therefore poles), $\alpha_\pm$ refers to the complex-conjugate pair nearest to or on the physical sheet. In this sense the approximation (1) to the full cut contribution does not depend on the details of the branch-point singularities in the j plane,[8] even though the detailed behavior of $\alpha_\pm$ does.[7]

We would like to make a few comments about the phase φ of the residue γ defined above. It was remarked in Ref. 8 that if the complex pair of poles was on the physical sheet, it was not possible to predict the phase of the effective residue. However, it was also noted that if the poles were on the unphysical sheet, and if the weight function of the cut were smooth in the neighborhood of $j=\alpha_R$, then the phase could be calculated in terms of other parameters [see Eq. (9) of Ref. 8]. If, on the other hand, the weight function is rapidly varying—for example, if it has a zero near $j=\alpha_R$—this argument fails. In this more general situation we may write the usual Mellin tranform (suppressing the signature factor) of the amplitude as

$$\int_{-\infty}^{\alpha_c} dj\,\frac{\mathrm{Im}f(t,j)s^j}{(j-\alpha_+)(j-\alpha_-)}=\int_{-\infty}^{\alpha_c} dj\sum_{n=0}^{\infty}a_n j^n\frac{s^j}{(j-\alpha_+)(j-\alpha_-)}$$
$$=\sum_{n=0}^{\infty}a_n\left(\frac{d}{d\ln s}\right)^n\int_{-\infty}^{\alpha_c}dj\,\frac{s^j}{(j-\alpha_+)(j-\alpha_-)},\tag{3}$$

assuming a power series expansion of the weight function, $\mathrm{Im}f(t,j)$. Over the range where the integral can, as in Ref. 8, be approximated by a pair of complex poles of the form $\beta_+s^{\alpha_+}+\beta_-s^{\alpha_-}$, the derivatives in the equation can be carried out to obtain

$$\int_{-\infty}^{\alpha_c} dj\,\frac{\mathrm{Im}f(t,j)s^j}{(j-\alpha_+)(j-\alpha_-)}\approx\mathrm{Im}f(t,\alpha_+)\beta_+s^{\alpha_+}+\mathrm{Im}f(t,\alpha_-)\beta_-s^{\alpha_-}.\tag{4}$$

But now the function $\mathrm{Im}f(t,\alpha_\pm)$ has a phase which is unknown so that the overall phase of the coefficient of s^{α_+} or s^{α_-} is unknown even though that of β_+ and β_- can be calculated.

Since we believe it to be quite reasonable for a rapid variation to occur in $\mathrm{Im}f$—for example, that $\mathrm{Im}f$ has a zero near $j=\alpha_R$—we have permitted the phases to be arbitrary. Thus the phase φ in the expression (2) above will be arbitrary.

An important fact to note about expression (1) above is that the signature factors in the two terms are not complex conjugates of each other while all other factors are. The phase of the amplitude, therefore, has additional contributions besides that from the usual signature factor which depend on $\mathrm{Im}\alpha$ and φ.

One also notes that spin-flip and spin-nonflip amplitudes will, in general, have different phases even though they may have the same $\alpha_\pm$. This is because the corresponding φ's will be different. Thus in such cases the polarization will not vanish. Moreover, the phase of the amplitude can be such as to make $\mathrm{Im}T$ vanish (the "crossover"-type zero) at a certain t value without requiring $\mathrm{Re}T$ or $|T|$ to vanish at the same point. Thus the difficulties with regard to factorization encountered in connection with the crossover phenomenon can be avoided.

We have used the above model to describe πN charge-exchange scattering where only ρ is exchanged. The relevant amplitudes here are $A'^{(-)}$ and $B^{(-)}$ which are parametrized as follows:

$$A'^{(-)}=\gamma_{+A}\frac{(1-e^{-i\pi\alpha_+})}{\sin\pi\alpha_+}\left(\frac{s}{s_0}\right)^{\alpha_+}+\gamma_{-A}\frac{(1-e^{-i\pi\alpha_-})}{\sin\pi\alpha_-}\left(\frac{s}{s_0}\right)^{\alpha_-},\tag{5a}$$

$$B^{(-)}=\gamma_{+B}\left(\frac{1-e^{-i\pi\alpha_+}}{\sin\pi\alpha_+}\right)\left(\frac{s}{s_0}\right)^{\alpha_+-1}+\gamma_{-B}\frac{(1-e^{-i\pi\alpha_-})}{\sin\pi\alpha_-}\left(\frac{s}{s_0}\right)^{\alpha_--1},\tag{5b}$$

$$\gamma_{+A}=h_0e^{h_1t}(\alpha_++1)e^{i\varphi_{+A}},\quad\gamma_{+B}=d_0e^{d_1t}\alpha_+(\alpha_++1)e^{i\varphi_{+B}}.\tag{5c}$$

For α and φ we consider two cases: In case (i),

$$\alpha_+=a+bt+ig(-t)^{1/2},\quad\varphi_{+A}=\pi(-t)^{1/2}(\gamma_0+\gamma_1t),\quad\varphi_{+B}=\pi(-t)^{1/2}(\lambda_0+\lambda_1t),\tag{6a}$$

while in case (ii),

$$\alpha_+ = a + bt + ig, \quad \varphi_{+A} = \pi(\gamma_0 + \gamma_1 t), \quad \varphi_{+B} = \pi(\lambda_0 + \lambda_1 t). \tag{6b}$$

Case (i) corresponds to a square-root singularity in $\alpha(t)$ in the t plane. For simplicity we have chosen $t=0$ to be the branch point. Case (ii) corresponds to a logarithmic singularity. These two types are perhaps the most representative of the types of singularities we may expect to encounter. [The singularity in the t plane of $\alpha(t)$, it may be noted, is related to the singularity in the j plane of the amplitude.]

We have obtained excellent fits to the πN charge-exchange differential cross section and polarization data, as can be seen from Figs. 1 and 2. Our fits for the total cross-section difference, $\sigma_t(\pi^- p) - \sigma_t(\pi^+ p)$, are also very good. However, the fits are unable to reproduce what appears to be a sharp break around $p_L = 10$ BeV/c. More accurate data are obviously needed. The data we have used are from Refs. 9, 10, and 11. We can obtain a zero in Im$A'^{(-)}$ (the crossover zero) but our results are insensitive to its precise location. It can be varied between $t=-0.15$ and -0.5 BeV^2 without significantly affecting the overall χ^2. Even though it is known experimentally that the elastic differential cross-section difference, $d\sigma(\pi^- p)/dt - d\sigma(\pi^+ p)/dt$, vanishes at $t \simeq -0.15$ BeV^2, this does not necessarily imply that Im$A'^{(-)}$ should vanish at precisely the same point since, unlike the previous (real pole) case, we have additional phases involved in our model. Thus one will have to re-examine the crossover phenomenon for elastic scattering with complex poles for P' and, perhaps, for P as well. For a zero in Im$A'^{(-)}$ at $t=-0.25$ BeV^2 the parameters are as follows. In case (i),

$a=0.53$, $b=1.02$ BeV^{-2}, $g=0.20$ BeV^{-1}, $h_0=0.65$ mb BeV, $h_1=-0.31$ BeV^{-2},

$\gamma_0=-1.40$ BeV^{-1}, $\gamma_1=-0.80$ BeV^{-3}, $d_0=27.2$ mb, $d_1=-0.49$ BeV^{-2},

$\lambda_0=-0.28$ BeV^{-1}, $\lambda_1=0.72$ BeV^{-3};

in case (ii),

$a=0.50$, $b=0.95$ BeV^{-2}, $g=0.088$, $h_0=-0.088$ mb BeV, $h_1=0.81$ BeV^{-2}, $\gamma_0=0.69$,

$\gamma_1=1.08$ BeV^{-2}, $d_0=28.8$ mb, $d_1=-0.82$ BeV^{-2}, $\lambda_0=-0.16$, $\lambda_1=0.66$ BeV^{-2}.

In all fits, the scaling parameter s_0, was fixed at 1 BeV^2.

Within experimental errors we are unable to say which of the above two cases is better. The most

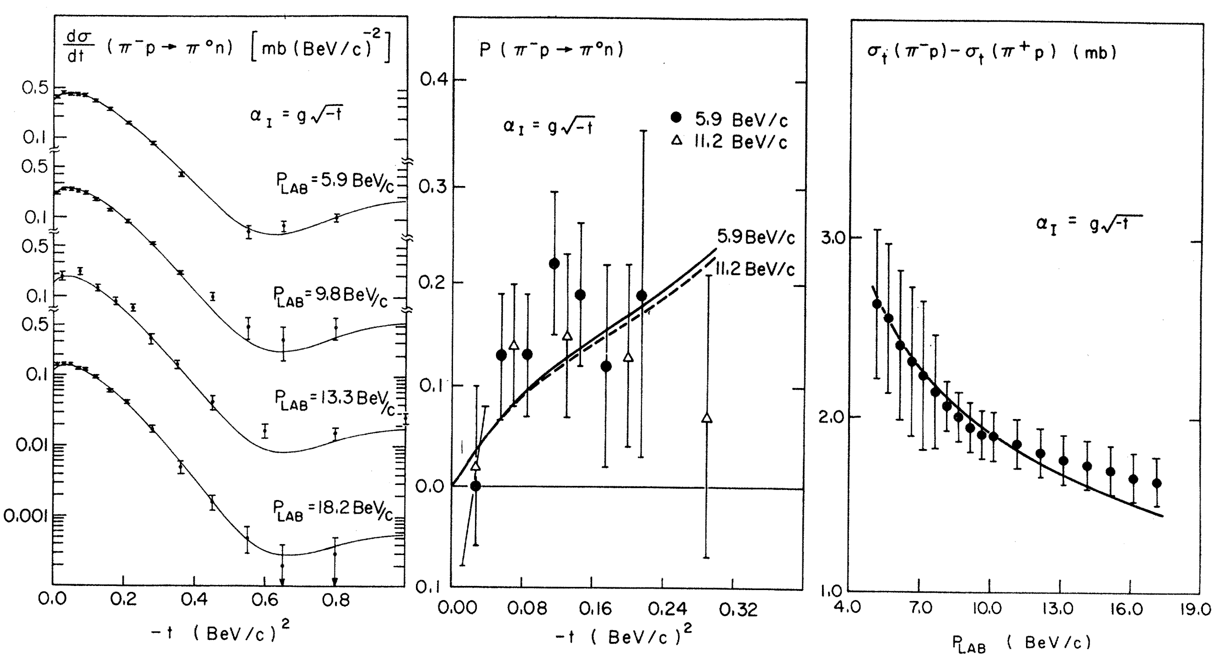

FIG. 1. Complex-conjugate Regge-pole fits to the πN charge-exchange differential cross section, polarization, and total cross-section difference $\sigma_t(\pi^- p) - \sigma_t(\pi^+ p)$, corresponding to case (i) of the text.

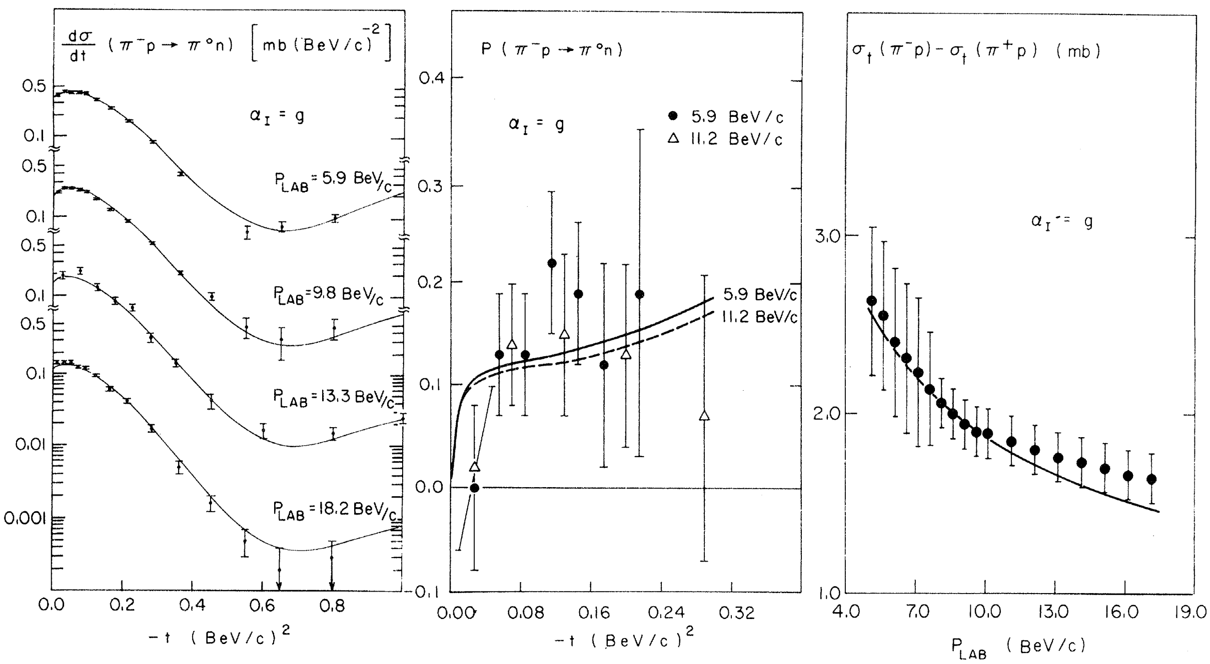

FIG. 2. Same as Fig. 1, but for case (ii) of the text.

characteristic difference in the two cases lies in their predictions for the polarization at very small $-t$. Case (i) gives the behavior $(-t)^{3/2}$, whereas case (ii) gives $(-t)^{1/2}$. Further experiments as well as other checks (such as finite-energy sum rules) may help in distinguishing the two cases.

The complex-pole approximation to the cut is presumably very good in the energy region under discussion and, therefore, it would be almost impossible to distinguish between the case where the Regge poles are on the physical sheet and when they are on the unphysical sheet. However, at extremely high energies where the approximation breaks down, the cut contribution will behave as $s^{\alpha_c}/(\ln s)^{\beta}$ and one may then be able to understand the situation better.

It is worth emphasizing, incidentally, that even if $\mathrm{Im}\alpha \to 0$, as is the case with the absorptive model, the energy dependence of the pole-cut combination is like s^{α_R}, in the moderate-energy region, not like s^{α_c} (where α_c is the branch point). This is so because even when the pole lies on the negative real j axis, the arguments used for complex poles in Ref. 8 are applicable here. Thus, earlier attempts to fit the data with Regge cuts behaving like s^{α_c} are not correct unless the energy is extremely high, regardless of the size of $\mathrm{Im}\alpha$.

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